Section 11.10 Solutions

14. $f(x) = x - x^3$ $\quad f(-2) = 6$

$f'(x) = 1 - 3x^2$ $\quad f'(-2) = -11$

$f''(x) = -6x$ $\quad f''(-2) = 12$

$f'''(x) = -6$ $\quad f'''(-2) = -6$

$f^{(4)}(x) = 0$ $\quad f^{(4)}(-2) = 0$

So $x - x^3 = 6 - 11(x+2) + \frac{12}{2!}(x+2)^2 - \frac{6}{3!}(x+2)^3$

$= \boxed{6 - 11(x+2) + 6(x+2)^2 - (x+2)^3}$ for all x, $\boxed{R = \infty}$

16. $f(x) = x^{-1}$ $\quad f(-3) = -\frac{1}{3}$

$f'(x) = -x^{-2}$ $\quad f'(-3) = -\frac{1}{9}$

$f''(x) = 2x^{-3}$ $\quad f''(-3) = -\frac{2}{27}$

$f'''(x) = -3 \cdot 2x^{-4}$ $\quad f'''(-3) = -\frac{3!}{3^4}$

$f^{(4)}(x) = 4 \cdot 3 \cdot 2x^{-5}$ $\quad f^{(4)}(-3) = -\frac{4!}{3^5}$

So $\frac{1}{x} = -\sum_{n=0}^{\infty} \frac{n!\,(x+3)^n}{3^{n+1}\,n!} = \boxed{-\sum_{n=0}^{\infty} \frac{(x+3)^n}{3^{n+1}}}$, $\boxed{R = 3}$

from the Ratio Test: $\lim_{n\to\infty} \left| \frac{(x+3)^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{(x+3)^n} \right| = \left| \frac{x+3}{3} \right| < 1$

18. $f(x) = \sin x$ $\quad f(\frac{\pi}{2}) = 1$

$f'(x) = \cos x$ $\quad f'(\pi/2) = 0$

$f''(x) = -\sin x$ $\quad f''(\pi/2) = -1$

$f'''(x) = -\cos x$ $\quad f'''(\pi/2) = 0$

$f^{(4)}(x) = \sin x$ $\quad f^{(4)}(\pi/2) = 1$

So $\sin x = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n (x - \frac{\pi}{2})^{2n}}{(2n)!}}$, $\boxed{R = \infty}$